Accessing and Dealing with Mathematics as a Blind Individual: State of the Art and Challenges

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ABSTRACT

Whereas the computer has been extremely helpful for blind individuals to cope with very many aspects of daily life, its support for managing Mathematics to the target group is still quite limited. This is a critical problem, because on the one hand Mathematics becomes more and more important for education, on the other hand blind people face tremendous problems in dealing with the subject. Information Technology has a high potential to support blind people in overcoming these problems, a potential that has not yet been fully exhausted.

The current paper, updating information from a publication by Archambault and others, will give an overview of the extent to which that potential is currently exploited, by summarizing the state of the art of IT to support the target group in Mathematics, with an emphasis on contributions delivered at our university. It also points out a series of open problems to be overcome by future research, describing challenges for improving the current situation.

1. INTRODUCTION

The basic problems faced by a blind person when dealing with Mathematics can be classified in three categories:

- Reading Mathematics – accessing mathematical information: As mathematical contents most of the time are present in printed form, and, if digitally available, in a two-dimensional, visually appealing rendering, ways have to be found to transform the information into a linear structure suited for a blind person's needs.
- Navigating within mathematical content - browsing: Mathematical formulae tend to deploy to a high level of complexity, even if they are quite short. Vision turns out to be a very powerful helper to overcome that complexity, but it is not available, or at least not fully available, to a blind or visually impaired person. We are therefore faced with the task of finding alternative methods which let a blind individual browse through a complex mathematical expression, or a whole calculation, without losing one's overview over the content.
- Doing Mathematics – manipulating contents: What was said before about navigation, is true to a much higher degree as soon as a blind person begins to actually calculate: Already at quite a low level of education, calculations tend to become lengthy, complex, and intricate; what is a challenge already for a sighted person – to organize one's calculation in a clear and consistent manner, to keep track of sub and side calculations, to manage the complexity of expressions -, turns out to be much more difficult if you cannot see; this is especially true because standard visual methods, like quickly jumping to a relevant spot within a formula, or marking a part of an expression as "already dealt with", are not easily applicable for a blind person.
- As we shall see in this survey, for all of these three basic problems there are solutions by now, but most of them are partial, not completely satisfactory, and not applicable to everyone in need of them. This is particularly true for the third main problem, doing Mathematics.

2. READING MATHEMATICAL DOCUMENTS

2.1 State of the Art

2.1.1 Two-dimensional (sighted) vs. Linear (blind) Representation:

The standard representation for mathematical contents in the sighted world is a two-dimensional, graphical one: Not only does it contain a wealth of special symbols, reaching far beyond roman or Greek letters, numbers, and operators, it also makes use of two-dimensional techniques like arranging characters in different heights above or below the base line in order to convey structure of a formula – common examples are indices, roots, and fractions.

Contrary to this, the two standard modalities used by blind people – synthetic speech and refreshable Braille output – cannot make use of two-dimensional arrangement techniques: Speech output, being strictly sequential, and Braille output, giving access to quite a short line of not more than 80 characters at a time, must be considered linear output modalities. In order to convey structure of a formula within such a linear representation, alternative methods need to be invented. In many cases, such methods of structuring make the output complex and difficult to read, and they are a reason why most linear codes, as will be described below, are quite hard to learn and to use.

2.1.2 Braille Codes:

Right after the traditional two-dimensional representation of mathematical formulae, Braille codes are the oldest way of representing Mathematics among those discussed in this survey: Already by the end of the 19th century, attempts were made to expand traditional Braille such that it would be able to represent mathematics as well. In the course of time, quite a number of mathematical Braille codes were defined [2,3].

Two basic problems are connected to these codes:

- Since almost every country invented their own code, we now have more than 50 mathematical Braille codes, which, although all of them build on traditional Braille, are quite different from each other – there is little evidence that attempts to unify them may succeed.
- Since Braille takes quite a lot of space, such codes tend to contain complex rules to convey mathematical structure in a space-saving fashion, which makes learning these codes difficult.

2.1.3 LaTeX Source:

This is a completely different approach: The typesetting program TeX, today most often used in terms of its macro package LaTeX, is out there for over 30 years now, and it is widely used in the mainstream to publish Mathematics, especially because of the premium quality visual output it produces. On the other hand, LaTeX source code is pure ASCII text, such that it may be used as a linear mathematical notation for blind people. For this reason, at least in theory, a mathematical document, once present in LaTeX, would be accessible to a blind user. Since, as stated above, quite a lot of mathematical literature nowadays is published in LaTeX, this, again in theory, would open a great amount of that literature to the target group.

We say "in theory", because in practice LaTeX source is hard to read, yes, it may become exceedingly difficult to read. This is for two reasons: First, LaTeX solves the problem of conveying structure in a linear modality by just enclosing elements belonging together in curly braces, which is a simple, but space-consuming approach. Second, real-world LaTeX comes with lots of formatting information which is of little meaning for the blind reader, but which desturbes the flow of reading, quite often so heavily that a document may get practically unreadable.

2.1.4 MathML:

MathML [4] is an XML application designed for representing mathematical contents. Especially in conjunction with XHTML, it gained great popularity, such that, as with LaTeX, a great amount of today's mathematical publications are present in that format. As said with LaTeX, MathML also is pure text, such that material present in that format would, in theory, be readily accessible to blind readers. However, what was said about LaTeX's difficult readability is true to a much larger extent for MathML – the XML tags used to convey structure require very much space on a Braille display, such that even very short and simple formulae would become tremendously long in MathML, which voids its readability for any human being, especially for a blind one.

2.1.5 Other ASCII Maths Codes:

LaTeX source and MathML may be viewed as ASCII Maths codes, because they consist of pure ASCII text. While both codes were never designed with blind readers in mind, there do exist some ASCII Maths codes which were exclusively designed for that target group, although they gained little importance. Examples are the AMS (ASCII Mathematikschrift) developed at the university of Karlsruhe, and the "Stuttgarter mathematikschrift".

2.1.6 The Conversion Problem: UMCL

From what was said so far, it follows that, among all the various codes introduced, the Braille codes will generally provide the best reading experience for a blind person. This being said, we see, however, the problem that very little mathematical content will be available in such a code – much more would be available in LaTeX or in MathML, which formats are both difficult to read. Also, we know about the problem that Braille codes are not compatible to each other, meaning that a reader who is familiar with one code will not automatically be able to read another one.

In light of these difficulties, a service would be desirable which converts mathematical contents from a popular, but badly readable format such as LaTeX or MathML into a better readable format of the user's choice, e.g., a Braille code with which (s)he is familiar. Also, a service to do the conversion the other way round would be desirable, for then a blind student could write an exercise in the Braille code of his/her choice and then convert it into LaTeX, from which a visually appealing version for the teacher could be derived.

Such a service exists: It is the UMCL – Universal maths Conversion Library, introduced by Dominique Arcambault from the university of Paris 8 [5,6]. It is a free library specification, according to which everyone is encouraged to contribute input or output modules. An input module from a specific Maths code, e.g., LaTeX, takes a document written in the said code, and converts it into the UMCL intermediate format. Contrary, an output module, e.g., for Marburg Braille code [2], takes a document written in UMCL intermediate format and converts it into the said code, in our example Marburg Braille.

The UMCL chose as its intermediate format a variant of MathML called "Canonical MathML" {7]. By this simple scheme, at least in theory, conversion from any maths code into any other one is possible, provided that input and output modules for the codes involved are available.

Among the codes for which currently input and output modules were provided, we have:

- LaTeX
- French Braille Code (in several variants)
- Marburg Braille Code (Germany)
- Italian Braille Code

• Nemeth Braille Code (USA)

2.1.7 The MathInBraille Web Service

A very convenient application of the UMCL is the MathInBraille web service [8,9]. This is an online service which lets one convert a text containing Mathematics in LaTeX or in MathML into text where the formulae are written in one of the Braille codes for which a UMCL output module exists.

Important is here that not only simple formulae, but whole text documents containing formulae may be converted. Such documents may either be LaTeX documents, or XHTML documents containing formulae written in MathML. In the first case, the TeX4HT converter [10] is called in order to translate the whole LaTeX document into an XHTML document with the formulae written in MathML, which then, by UMCL, is translated to the Braille code chosen by the user. In the second case, the first step, the invocation of TeX4HT, can be omitted – UMCL is called directly.

2.1.8 Speech and Sound Based Access

So far we only talked about Braille representation of Mathematics. Already in the 1990's, attempts were made to make Mathematics accessible via speech and sound – the most important product was ASTER by T. V. Raman [11]. In 2010, Fitzpatrick undertook research on the question how prosody in synthetic speech might be a contribution to convey structure within a sequentially verbalized formula [12].

Nowadays, attempts to express MathML, rendered in a web browser for sighted people, via synthetic speech are undertaken. Important is MathPlayer by Design Science [13], a plugin to the Microsoft Internet Explorer. Also, MathML rendered by a web browser is now verbalized both when the Safari browser is used on an IOS device with the VoiceOver screen reader, and when the content is browsed by Google Chrome, and the ChromeVox browser extension [14] is used.

2.1.9 Mathematical OCR – InftyReader

Whereas more and more mathematical content becomes digitally available, the vast majority of that content is still present only on paper. To make this accessible, OCR technology is needed. The InftyReader, from Suzuki and others [15], is the world's first, and only, solution to tackle this. While it may, under the right circumstances, do a good OCR job and present the contents accurately and correctly in LaTeX source or in MathML, readability of the output is often difficult, such that intense post-processing is needed.

2.2 Challenges

Although, as reported, more and more mathematical documents become accessible, there is still much to do: The problem is almost always human readability, arising mainly from the fact that visual representation, which is mostly the source which conversion tools need to rely on, does not fully convey the structure, the semantics, of a mathematical expression, such that heuristics is often needed.

As for the MathInBraille web service, we at the University of Linz currently undertake efforts to improve stability and accuracy of the conversion process, by building a post-processor for the TeX4HT converter. A post-processor is also being developed for InftyReader, the mathematical OCR engine.

3. NAVIGATING THROUGH MATHEMATICAL CONTENTS - BROWSING

3.1 State of the Art

3.1.1 The Problem of Complexity in Navigation

Even if we do have a mathematical document accessible in a well-readable code which we are familiar with, reading, more precisely, understanding, the content will most likely remain difficult. This is because it takes much time, and effort, to traverse the overall structure of an expression, which is typically a tree, a process in which sight would substantially help, but sight is not, or not fully, available to our target group.

It is for this reason that recently several software tools were devised to support a blind person in navigating through an expression. In the one or the other sense, each of these tools grants "collapse and expand" support, as is well known from modern programming environments.

3.1.2 Speech-based Solutions – Math Genie etc.

Probably the first product in this direction was the Math Genie by Arthur Karshmer and others [16]. This speech-based tool was able to read a formula in MathML and allowed navigation through its structural tree, where children of a node could be hidden and displayed again on command.

3.1.3 The Problem of Collaboration/Synchronization – First MAWEN Prototype

When a blind individual is able to traverse a mathematical expression, (s)he will want to let his/her sighted peers, or sighted teacher, take part in the experience – at least (s)he wants to show a spot in a formula s(h)e is currently reading to sighted people around him/her. This collaborative scenario, which is of special importance in a classroom setting, is also important the other way round: A sighted teacher shows a spot of a formula to his/her students, and a blind student wants to be able to follow it through his/her special technology.

It is exactly this which was reached through a prototype / demonstrator, developed within the FP7 EU-funded project MICOLE [17, 18].

3.1.4 A New Approach: Gesture-based Browsing

For quite a long time, it was implicitly clear that, when a blind person would input Mathematics to the computer, (s)he would do it through the computer's keyboard. But this is not the only possibility: Gestures, as they are common on modern mobile devices, or even movement through a room, movement of the head etc. might be equally feasible input modalities. Touch gestures are considered of special interest, because they would enable blind pupils to do Mathematics with their mobile devices, which gain ever increasing popularity among the target group.

In order to answer the research question whether input by touch gestures were feasible for blind individuals, a master thesis at the University of Linz was undertaken by [19]. She could answer the question positively, by developing a prototype, which lets a user traverse the tree of a document, or that of a mathematical formula, by touch gestures. As output modality, synthetic speech was chosen. As document format, documents in the computer algebra system Mathematica, so-called "notebooks", were chosen; in fact, the whole prototype is written purely in Mathematica.

3.2 Challenges

None of the navigation tools introduced is really sufficiently well developed to be used in practice. The collaborative tool developed within Micole needs substantial improvements in terms of performance and reliability. The gesture-based navigation tool functions in principle, but it needs to be adapted to more input devices than just the touch pad of a laptop PC, and its usability needs decisive improvement.

4. DOING MATHEMATICS

4.1 State of the Art

4.1.1 The Complexity Problem: Keeping an Overview over a Calculation

When actually doing Mathematics, new problems arise in addition to those of reading and navigating: Within a calculation, even within a comparatively simple one, you need to jump to different spots in side calculations, need to jump back to the main calculation, and remember items, e.g., intermediate results, from side calculations. The art of organizing a calculation such that an overview of it may be kept easily is a complex task imposing a considerable challenge to everyone, especially to someone without or with no full sight. The problem was analyzed in detail in [20].

4.1.2 Calculation Templates/Wizards: MAWEN Prototype 2

Within the already mentioned MICOLE project, a second MAWEN prototype was developed [20], in order to demonstrate how software may support a student in doing a calculation. By several concrete examples from elementary arithmetics and from algebra, so-called "assistants", or "wizards", were designed, which guide the student through the calculation and take the burden of remembering expressions from him/her.

4.1.3 Math Editors

Over the past few years, special math editors for blind people were developed in order to support them in writing expressions: Instead of typing complicated code like with LaTeX or MathML, such an editor typically presents a menu, or tool bar, offering mathematical symbols and structures organized in categories.

- MathType: This is a Microsoft Word addin from Design Science [21], mainly developed for the mainstream user: You get a menu or toolbar offering mathematical symbols or structures to input, which are shown in a visually appealing way within the document edited.
  This approach, though inaccessible, is mentioned here because there is an option to convert the whole active document into LaTeX source. This code, when edited, may be converted back into original MathType rendering, which, in theory, would furnish a fine collaborative environment between sighed and blind users. We say "in theory", because in practice MathType documents come with lots of formatting information, which will be converted one-to-one to LaTeX, which again cuts down readability.
- Lean Math: This is an auxiliary program for MathType which, on the one hand, presents a MathType document in a form adapted to synthetic speech and Braille users, and, on the other hand, replaces the inaccessible toolbars to input Mathematics by an accessible alternative [22].
- ChattyInfty: This math editor, developed by the Infty group as a supplementary product to the above-mentioned InftyReader, does similar things as Lean, but without relying on MathType – it holds the mathematical information input by the user in a proprietary, though XML-based format; it has import and export capabilities with various formats such as LaTeX, MathML, and a special format called Human Readable TeX, which is meant to improve readability while still similar to the original TeX format [23].

4.1.4 Tools to Deal with Algebra

Within a PHD thesis supervised by Dominique Archambault at the University of Paris, Silvia Fajardo Flores developed a software prototype to support a student in doing elementary algebraic tasks [24,25].

4.2 Challenges

A fully-fledged, practically usable, and optimized Mathematical Working Environment – MAWEN - is still to be developed. Except from the Math editors, which are quite complete solutions by now, all the tools introduced here do not reach beyond prototype level. In particular, the MAWEN wizards developed within the MICOLE project need to be made much more configurable, in that they should give the teacher or student control which aspects of a calculation they should support.

A further challenge would be the integration of support as described in this survey into an off-the-shelf computer algebra system such as Mathematica – when considering the touch gesture input prototype one can see the benefits of such an approach.

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